Quantized tension:
Stringy amplitudes with Regge poles and parton behavior

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Abstract

We propose stringy hadronic amplitudes that combine some of the features of sister trajectories and running tension. By summing over string amplitudes with varying Regge trajectories that have integer tension and converging intercept, we obtain parton hard-scattering and Regge soft-scattering behaviors, while preserving discrete poles in both momentum and angular momentum.

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1 Introduction

Hadronic physics can be divided into four regions of "phase space": (1) low energy, (2) spectrum, (3) high energy, small angle, and (4) high energy, large angle. Low energy (including the low end of the spectrum) is described by many methods, such as lattice Quantum ChromoDynamics, nonlinear sigma models, instantons, and nonrelativistic quark models (which also handle mass differences within any multiplet). The parton model, and more accurately perturbative QCD, describe high energy at large angles, and to a lesser extent at small angles (total cross sections and related processes). Regge theory describes the spectrum, and scattering for high energies at small angles, as well as being consistent with low energy.

Regge theory directly relates the hadronic spectrum to the high-energy behavior of scattering amplitudes at small angles.[1] Amplitudes are described by a Regge trajectory $\alpha(t)$: The spectrum consists of states of spin $J = \alpha(M^2)$ at mass M, while amplitudes go as $s^{\alpha(t)}$. The requirement of a perturbation expansion whose lowest order realizes this behavior only as poles in angular

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[1]For a review, see [1].

momentum implies [2] Dolen-Horn-Schmid duality [3], which is explicitly realized in string theory. Experiment has verified duality qualitatively, and Regge high-energy behavior up to $|t|$ of the order of 1 GeV^2, but the most striking confirmation of Regge behavior is the appearance of the known hadrons on very linear trajectories up to spins as high as 4.

From the beginning it was known that string amplitudes had exponential decay at large transverse momenta, as seen in the fixed-angle limit of high-energy scattering [4, 5], and it was soon realized that this could not easily be reconciled with the observed power-law behavior described by parton models [6]. One interpretation was that strings and QCD were dual descriptions of the same physics, so that parton behavior in theories of hadronic strings is nonperturbative, just as confinement is nonperturbative in QCD. Thus at the very least an infinite summation of diagrams would be required to obtain one description from the other. For example, in Regge theory a cut produced from the exchange of multiple reggeons has a Regge slope a fraction of that of the original reggeon [1], so a summation can produce an effective leading trajectory, running along the tops of the trajectories of the pole and cuts, that has harder behavior in the appropriate region [7]. A similar approach is to use the "sister trajectory" poles related to these cuts, also found in progressively higher-point amplitudes [8]. Unfortunately such an approach is intractable, just as trying to calculate the soft parts of perturbative QCD amplitudes by infinite resummation of QCD graphs. Also, the hard limits obtained in both the cut and sister approaches are not the usual power laws of the parton model.

An alternative method is to use the coordinate of a fifth dimension as an effective running tension [9]. In fact, this can be realized along the lines of the Anti-deSitter/Conformal Field Theory conjecture [10]. The simplest models are built from old-fashioned Veneziano or Virasoro-Shapiro amplitudes A_n integrated over the tension with an appropriate weight factor as [11, 12]^2

$$\hat{A}_n(p_1, \ldots, p_n; \xi_i, \ldots, \xi_n) = \int_{r_0}^{\infty} dr \ r^{3-\Delta_n} A_n(p_1, \ldots, p_n; \xi_i, \ldots, \xi_n)|_{\alpha' \to \alpha' R^2 / r^2} \quad , \qquad (1.1)$$

where p_i's and ξ_i's are momenta and wavefunctions of particles. Δ_n, R and r_0 are some parameters whose meaning will be clarified shortly.^{3} We will call A_n (without any modification of α') a "primary" amplitude to differentiate it from $\hat{A}_n$.

Integrating from zero to infinity manifestly produces a scale invariant amplitude, appropriate to some conformal field theory (for example, N=4 super Yang-Mills), while putting a lower limit on the integration keeps that limit as a unit of tension, breaking scale invariance, as appropriate to QCD. This effectively produces a continuum of sister trajectories, but all for the four-point amplitude (for example). The "top" trajectory for positive argument corresponds to the minimum-tension trajectory, while for negative argument it corresponds to the infinite-tension (zero-slope) trajectory, appropriate to a parton. Regge high-energy behavior comes from the smooth joining region intermediate between these two top pieces.^{4}

Unfortunately, the "smearing" of the trajectories replaces the particle poles with cuts (as opposed to the usual distinct poles plus cuts required by unitarity): Effectively this means that

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^{2}If one assumes that perturbation theory is a topological expansion, then the full amplitude can be defined as $\hat{A}_n(p_1, \ldots, p_n; \xi_i, \ldots, \xi_n) = \sum_{\chi=2}^{-\infty} g^{-\chi} \hat{A}_n^{(\chi)}(p_1, \ldots, p_n; \xi_i, \ldots, \xi_n)$, where χ is the Euler number [13].

^{3}Note that one can think of $\hat{A}_n$ as the Mellin transform of A_n, namely $\hat{A}_n = r_0^{4-\Delta_n} \int_1^{\infty} d\rho \, \rho^{-\omega-1} A_n|_{\alpha' \to \hat{\alpha}/\rho^2}$, with $\omega = \Delta_n - 4$ and $\hat{\alpha} = \alpha' R^2 / r_0^2$. The factor $r_0^{4-\Delta_n}$ simply adjusts the dimension.

^{4}For a different approach to Regge physics also motivated by the AdS/CFT conjecture, see [14] and references therein.

for any spin the masses are continuous. Similar behavior occurs in conformal theories with nonvanishing mass: A conformal transformation scales p^2, and thus the mass, so it is possible for massive theories to be conformally invariant if the mass spectrum includes all positive real numbers. In this case not all continuous masses extend to zero once one has introduced the QCD tension as an integration limit, breaking the conformal invariance but leaving the continuous mass problem unresolved. (Continuous mass is a possible problem in AdS/CFT, and related problems appear in membranes and subcritical closed strings.) In particular, this destroys the usual low-energy limit ("pion physics").

In this paper we propose amplitudes that simultaneously have (1) a discrete (integer-spaced) particle spectrum appearing on linear Regge trajectories[5], (2) Regge behavior in the soft limit, and (3) parton behavior in the hard limit. Continuous spectra are avoided by replacing the integral over tension with a sum. The original spectrum is preserved by requiring the tensions to be integer multiples of the original, as for sister trajectories. The correct parton behavior follows from requiring that the trajectory intercepts (which also have a type of integer constraint) converge. (For large integers, which contribute to the parton behavior, the sum can be approximated as an integral.) We do not provide a string Lagrangian for these amplitudes, but propose them as a starting point, as was the case for the original string.

We propose these amplitudes to describe "tree-level" behavior with respect to both partons and hadrons, which seems the only way to perturbatively calculate amplitudes that necessarily contain both hard and soft pieces. Generally, when nonperturbative properties are important in a formulation of a theory, it is an indication of the limitation of that formulation. For example, in the usual formulation of Quantum ElectroDynamics, one first calculates classical (tree), then perturbative (loop) contributions, and that is sufficient for observed phenomena (except when corrections of a non-electrodynamical nature contribute). On the other hand, in QCD nonperturbative effects such as renormalons or confinement are important in almost all processes. A more useful alternative would be a formulation where both confinement and partons are incorporated at "tree" level, as defined by partons or hadrons appearing only as poles, with small corrections from loops, without the need for further contributions that are both poorly defined and almost impossible to calculate. There is some experimental evidence to indicate that such an approach is possible [16–18]. In this paper we propose such a model, and give some preliminary comparison to the real world.

As we consider only the tree contribution, we compare mostly to reggeons, since for the pomeron ("glueballs") cuts are difficult to disentangle from poles for $t < 0$, while no glueballs have been unambiguously identified to allow identification of trajectories for $t > 0$.

2   Cuts and sisters

In this section we give some background on the cut and sister trajectory approaches to high-energy behavior to make the further discussion more tangible. In both the approaches, the main idea is that a resummation of string loops will modify the effective leading trajectory: In

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[5]In the literature there exist models also providing discrete spectra (see [15] and references therein). Their crucial differences from ours are: (1) the use of the supergravity approximation, and (2) occurrence of continuous spectra as well as poles. Since their assumptions do not seem fundamentally different from those of [9, 11, 12] (slight modification of the metric to implement a cutoff in the fifth dimension), in general it is difficult to see how this discrete spectrum can be made consistent with Regge behavior and allowed kinematic regions.

the cut approach one looks at the trajectories of the cuts produced by exchanges of multiple poles; similarly, in the sister trajectory approach, the sister trajectories appear only in higher-and-higher-point amplitudes, so a summation over these trajectories can be applied only by a summation over all loops.

In the case of cuts, the general rule for linear trajectories (poles or cuts), written as

$$\alpha(t) = \frac{\alpha'}{a} t - b + 1 \tag{2.1}$$

is that the trajectory α resulting from the exchange of multiple trajectories α_i satisfies

$$a = \sum a_i \quad , \quad b = \sum b_i \quad . \tag{2.2}$$

For example, if we consider the sum of n identical trajectories (in units $a_i = 1$), we find

$$\alpha_n(t) = \frac{\alpha'}{n} t - bn + 1 \quad . \tag{2.3}$$

(We could also consider one reggeon trajectory plus $n-1$ pomeron trajectories, with qualitatively similar results.) Sister trajectories have a similar form,

$$\alpha_n(t) = \frac{1}{n} \big(\alpha' t + \alpha_0\big) - \frac{1}{2}(n-1) \quad . \tag{2.4}$$

The basic idea is that in general one trajectory will be higher than the rest, where the value of n for that trajectory will depend on the value of t. Explicitly, if we sum the high-energy contributions over n,

$$\sum_n \beta_n(t) \big(\alpha_n(s)\big)^{\alpha_n(t)} \quad , \tag{2.5}$$

then the leading contribution can be found by a saddle-point approximation on the exponent of $\alpha_n(s)$, approximating the sum as an integral.[6] The result of treating n as continuous is that a differentiable curve is obtained for this effective trajectory, which is a better approximation than the piecewise differentiable trajectory that would be obtained by simply connecting together the pieces of whichever trajectory happens to be highest in any particular region. For a generic contribution that includes the cut and sister cases,

$$\alpha_n(t) = \frac{\alpha'}{n} \big(t - t_0\big) - bn + J_0 \quad , \tag{2.6}$$

where $b \geq 0$, we find the maximum (for $t < t_0$) from

$$0 = \frac{\partial}{\partial n} \alpha_n(t) = -\frac{\alpha'(t - t_0)}{n^2} - b \quad \Rightarrow \quad n_0 = \sqrt{\alpha'(t_0 - t)/b} \tag{2.7}$$

$$\Rightarrow \quad \tilde{\alpha}(t) = \begin{cases} \alpha_1(t) = \alpha'(t - t_0) - b + J_0 & \text{for} \quad t \geq t_0 - b \\ \alpha_{n_0}(t) = -2\sqrt{\alpha' b(t_0 - t)} + J_0 & \text{for} \quad t \leq t_0 - b \end{cases} \quad , \tag{2.8}$$

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[6]A better approximation would include the effect of β_n and the n-dependence of $\alpha_n(s)$, as we'll see in the following section.

where $\tilde{\alpha}$ is the "top trajectory" obtained by combining the parts of the trajectories from each n where it is greater than the others. This modifies the effective behavior of the amplitude, but not enough to mimic parton behavior in the hard limit. The only exception is the case $b = 0$: This is irrelevant to the sister case, while in the cut case it relates to the pomeron, whose intercept is near 1, with trajectories converging at $t = 0$. That case is too extreme, since it would eliminate Regge behavior altogether (flat trajectory for all $t < 0$).

3   New models

Our models will be based on several assumptions:

(i) *Amplitudes are sums of "standard" (primary) string amplitudes.* In this paper we examine only the 4-point amplitudes, so this means Beta functions, or more general ratios of products of Gamma functions, whose arguments are linear trajectories. This guarantees duality.

(ii) *All amplitudes have poles that are a subset of those of the "first" amplitude.* This prevents cuts in this Born approximation (unlike continuously running tension). The trajectories are then quantized, so we parameterize them by a positive integer "n", with "first" meaning $n = 1$. For simplicity we assume no degeneracy, so this one parameter is sufficient to identify a trajectory. (In principle, degeneracy might be hidden in the normalization of the weights.)

(iii) *The trajectories converge toward a flat trajectory.* This allows parton behavior [19], since reggeons with small slopes resemble ordinary particles for a large range of energy. The natural ordering is for the trajectories' slopes to decrease with increasing n. For simplicity we assume the slopes are non-degenerate. Thus the slopes approach zero in the limit as n goes to infinity. The intercepts also converge (unlike methods using cuts or the usual sister trajectories), so the "top trajectory" approaches a constant at $t = -\infty$. The use of an infinite number of trajectories is also a simplifying assumption, since it allows the small-slope contribution to be approximated by an integral: Integrating over slope (tension) produces approximate conformal invariance at large transverse momenta.

(iv) *The weights are also n-dependent, in a way consistent with quark counting rules in the hard scattering limit where they are relevant.*

To preserve the integer (in units $\alpha' = 1$) spacing of the poles, we require the masses of the states on the leading (linear) trajectory for each n satisfy

$$\alpha' M^2 + \alpha_0 = a_n J + b_n \quad \text{for mass } M \text{ and spin } J \quad , \tag{3.1}$$

where α_0 is an n-independent constant (determined by the trajectory for $n = 1$), and a_n and b_n are n-dependent integers. Since these states appear at integer $J = \alpha(M^2)$, we have for the trajectories

$$\alpha_n(t) = \frac{1}{a_n}\big(\alpha' t + \alpha_0 - b_n\big) \quad , \tag{3.2}$$

where a_n increases with increasing n. (We can always normalize α_0 so $b_1 = 0$; in most cases we also have $a_1 = 1$, so α_0 is the intercept for $\alpha_1(t)$.) Since the trajectories converge

$$\lim_{n \to \infty} \frac{b_n}{a_n} = \text{const} \tag{3.3}$$

power-law behavior in the hard scattering limit can be obtained by choosing the relative normalization of the weights for the amplitude $\hat{A}$ so that

$$\hat{A} = \sum_{n=1}^{\infty} w_n \, A(n) \quad , \tag{3.4}$$

where w_n is a weight and $A(n)$ differ only by the fact that they depend on α_n. One convenient choice is to take w_n in the form[7]

$$w_n = \frac{c}{n} \, a_n^{-c} \quad , \tag{3.5}$$

where c is some parameter, since this form is less sensitive to choice of a_n: When we approximate the sum as an integral for large n, if we choose a_n to go as a power of n in that limit, then different choices of that power will have little affect on the integral, as it depends on n only through a_n, b_n, and the "measure" dn/n. From a hard scattering analysis, c will turn out to be integer, half the total number of quarks. ($c = 4$ for the 4-meson amplitude on which we focus.)

4 Backgrounds

We have not given the physical interpretation of the integer parameter n. One possibility is first-quantization: It might be the zero-mode (perhaps the only mode) of a fifth dimension, whose momentum is compact.[8] If so, it would be interesting to see a relation to the model of [20] where the extra dimension is also latticized. Alternatively, it might be a consequence of latticization of the worldsheet itself: Random lattice quantization, before taking the worldsheet continuum limit, can lead to quantized values of the slope [21].

Another possibility is second-quantization: The slope, intercept, and string coupling are quantized, suggesting something along the line of the quantization of the gravitational constant found in another context by Bagger and Witten [22]. In this interpretation summation over the values of these couplings, like a sum over instantons, would initially be considered nonperturbative; the result of this simple resummation would then be treated as the tree approximation of a new perturbation expansion. The definitions of "nonperturbative" and "tree" are mere semantics; what matters is that our definition of "tree" gives a simple amplitude that one can apply explicitly.

Each of the integers a_n, b_n, and c would then be associated with the quantization of the "vacuum" value of a closed-string field: The slope (associated with the integers a_n) with the (four-dimensional) "graviton", the intercept (associated with the ratio b_n/a_n) perhaps with the "tachyon", and c, which is required by dimensional analysis in terms of the number of quarks, with some other scalar, like the "dilaton" (which by definition is related to dilatations and thus engineering dimension) or a higher-dimensional component of the metric. (Of course, for the pomeron, or closed hadronic string, all these states are now massive; we simply use the names associated with these fields in conventional string theory.)

Since a_n and b_n are functions of n, the first step would be to find background fields, representing a "ground state" solution of some field equations about which string perturbation is

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[7]Note that any function whose large-n asymptotics has such a form provides the parton behavior in the hard scattering limit.

[8]A topological quantity such as worldsheet instanton number or Euler number does not seem possible, since those are worldsheet-integrated quantities.

performed, that are functions of a fifth dimension r such that the fields take integer values when the "warp factor" $a(r)$ does: For example, it appears in the spacetime metric, and thus the string Lagrangian, as

$$ds^2 = -a(r)dx^2 + ... \quad \Rightarrow \quad L = \frac{a(r)}{\alpha'_{10}}(\partial x)^2 + \cdots \tag{4.1}$$

where α'_{10} is the usual slope of the 10D string. (We discard terms for other coordinates by assuming that they are x-independent.) The amplitude is then defined by

$$\hat{A}_4(\alpha') = \int_{r_0}^{\infty} \frac{dr}{r}\, ca(r)^{-c}\, A_4\left(\frac{\alpha'_{10}}{a(r)}\right) \quad , \quad \alpha' = \frac{\alpha'_{10}}{a(r_0)} \quad . \tag{4.2}$$

For example, the case of AdS_5 [11] in Eq.(1.1) has $a(r) = r^2/R^2$, where R is the radius of AdS_5. As a result, all parameters in the 10D formulation (α'_{10}, r_0, R, 10D string coupling) are replaced by just the 4D string coupling and slope α'.

The next step would be to replace this integral with the sum in Eq.(3.4) by performing the quantization

$$\frac{a[r(n)]}{a(r_0)} = \frac{a_n}{a_1} \quad , \tag{4.3}$$

where $r = r_0$ corresponds to $n = 1$. However, the dependence of the fields on r should be consistent with the relations between a_n and b_n that we have already found. Effectively, this is the same as looking at the large-n limit of our models, and treating the primary amplitudes as functions of continuous n (i.e., r) in this limit. This limit will also be important below in analyzing the high-energy limits of amplitudes.

This restriction eliminates certain types of backgrounds found in many supergravity solutions: For instance, in the D3 brane solution there is the nonconformal geometry

$$a(r) = \sqrt[k']{c_0 + c_k\left(\frac{r}{R}\right)^k} \quad , \tag{4.4}$$

with some integers k and k' ($\neq 1$), which will not lead to integer a_n. Also, we have not included the harmonic functions usually occurring in supergravity solutions. A true string derivation of our models will require understanding the origin of both background and quantization.

We have not taken into account the affects of (broken) supersymmetry: In particular, we have not considered Ramond-Ramond background fields, which would require a Green-Schwarz formulation.

5 Regge limit

5.1 Lowest order approximation

As a simple example, we apply the analysis of section 2 to a trajectory

$$\alpha_n(t) = \frac{\alpha'}{a_n}(t - t_0) + J_0 \quad , \tag{5.1}$$

for some integer J_0. The leading intercept takes the arbitrary value $J_0 - \alpha' t_0/a_1$, but for large n the intercepts converge to J_0.[9] Since by assumption a_n being positive increases indefinitely as n increases indefinitely, we easily obtain

$$\tilde{\alpha}(t) = \begin{cases} \alpha_1(t) = \frac{\alpha'}{a_1}(t - t_0) + J_0 & \text{for} \quad t \geq t_0 \ , \\ \alpha_\infty(t) = J_0 & \text{for} \quad t \leq t_0 \ . \end{cases} \tag{5.2}$$

We can thus arbitrary fit the point (t_0, J_0) where $\tilde{\alpha}$ goes from flat to slope 1. A particularly simple case is $a_n = n$.

By a slight generalization of the case $a_n = n^2$, the asymptotic intercept can be generalized to half-integer, and the top trajectory can be made more smooth:

$$\alpha_n(t) = \frac{\alpha'}{n^2}(t - t_0) + \frac{1}{n}\left(J_0 - \frac{1}{2}J_1\right) + \frac{1}{2}J_1 \quad , \tag{5.3}$$

where J_0 and J_1 are some integers obeying $J_1 \leq 2J_0$. The top trajectory is then given by

$$\tilde{\alpha}(t) = \begin{cases} \alpha_1(t) = \alpha'(t - t_0) + J_0 & \text{for} \quad t \geq t_0 - \frac{1}{2}(J_0 - \frac{1}{2}J_1) \ , \\ \alpha_{n_0}(t) = (J_0 - \frac{1}{2}J_1)^2/4\alpha'(t_0 - t) + \frac{1}{2}J_1 & \text{for} \quad t \leq t_0 - \frac{1}{2}(J_0 - \frac{1}{2}J_1) \ , \end{cases} \tag{5.4}$$

which replaces the flat part with a hyperbola. The extra parameter over the previous case allows for choice of the sharpness of the hyperbola, which allows a smoother transition to flatness. (The previous top trajectory is obtained for $J_1 = 2J_0$.)

As a further generalization, consider

$$\alpha_n(t) = \frac{1}{n^k}\left(\alpha' t + \alpha_0 - P_k\right) \quad . \tag{5.5}$$

of which the previous example is the special case $k = 2$. Its top trajectory is given by

$$\tilde{\alpha}(t) = \begin{cases} \alpha_1(t) = \alpha' t + \alpha_0 - P_k(1) & \text{for} \quad n_0 \leq 1 \ , \\ \alpha_{n_0}(t) = -P'_k(n_0)/k n_0^{k-1} & \text{for} \quad n_0 \geq 1 \ , \end{cases} \tag{5.6}$$

where P_k is a polynomial of degree k with positive coefficients. n_0 is a solution of the equation $\alpha' t + \alpha_0 = P_k - \frac{n}{k}P'_k$. Since the right hand side of this equation increases with increasing n for $n > 0$, the solution exists if $P_k(0) < \alpha' t + \alpha_0$. Note that other exactly solvable examples are those of $k = 3$ and $k = 4$.

The story becomes more and more involved when effects of β_n and $\alpha_n(s)$ are taken into account. The novelty is that n_0 depends on s in a way that restricts the Regge behavior to special kinematical regions. On the technical side, a difficulty is related to the problem of solving the equation for the top trajectory. The example of [11] includes simple power functions for w_n and a_n.

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[9]This includes as a special case the model of [11], where $t_0 = 0$, and $J_0 = 2$ is the usual closed string intercept, for each trajectory.

5.2   Continuous limit

The approximation of simply determining the top trajectory works well for values of t where $\tilde{\alpha}(t) = \alpha_1(t)$. In general this means for positive t, where the trajectories are fit to the spectrum, but can be extended some distance to negative t (e.g., by choice of the parameter t_0 in the above examples).

Since experiments have not yet determined dependence on t for a large range of negative values (in comparison with that for positive t), that may be sufficient. However, if we anticipate restrictions on possible models from criteria we have not yet analyzed (higher-point functions, loops, etc.), it will be useful to generalize by considering corrections to Regge behavior from n-dependence of the couplings that weight the primary amplitudes. This was found to be the case in [11], where the flat part of the top trajectory found in the first example above was found to have effective nonvanishing slope for a region consistent with experiment.

For the first model of the previous subsection, using the couplings of (3.5) with Veneziano amplitudes as the primary amplitudes,[10]

$$\hat{A} = \sum_{n=1}^{\infty} \frac{c}{n} a_n^{-c} \int_0^1 du \ u^{S/a_n + k} (1 - u)^{T/a_n + k} \quad . \tag{5.7}$$

Here $S = -\alpha'(s - t_0)$, $T = -\alpha'(t - t_0)$, $k = J_0 - 1$. We first replace the sum with an integral, and make the change of variables

$$v = \frac{1}{a_n} \quad . \tag{5.8}$$

If we assume a_n goes as a power of n (which we can normalize as $a_1 = 1$), then, dropping an overall constant,

$$\hat{A} \approx \int_0^1 dv \ cv^{c-1} \int_0^1 du \ u^{Sv + k} (1 - u)^{Tv + k} \quad . \tag{5.9}$$

Since we are looking for Regge behavior with nonvanishing slope, we will assume that for large s the integral over v is dominated by $v \approx 1$ (small n), and see under what conditions this assumption is justified.[11] We therefore rearrange this integral as

$$\hat{A} \approx \int_0^1 dv \ cv^{c-1} \int_0^1 du \ u^{S+k} (1 - u)^{T+k} e^{-(1-v)[S \ ln \ u + T \ ln(1-u)]} \quad . \tag{5.10}$$

Since S and T are linear in α', the exponential can conveniently be rewritten in terms of derivatives with respect to α', allowing the u and v integrals to be separated. The u integral can then be identified as the Veneziano amplitude for the first primary amplitude, yielding the expression

$$\hat{A} \approx f\left(\alpha' \frac{\partial}{\partial \alpha'}\right) B(-\alpha_1(s), -\alpha_1(t)) \quad , \tag{5.11}$$

with

$$f(x) = c \int_0^1 dv \ v^{c-1} e^{-(1-v)x} = \sum_{n=0}^{\infty} \frac{c!}{(c+n)!} (-x)^n = 1 - \frac{1}{c+1} x + O(x^2) \quad . \tag{5.12}$$

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[10]We can also include kinematic factors, which we assume are n-independent.

[11]See also [23].

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After taking the Regge limit, we have

$$\hat{A} \approx f\left(\alpha'\frac{\partial}{\partial\alpha'}\right)\Gamma(-\alpha_1(t))(-\alpha_1(s))^{\alpha_1(t)} \quad . \tag{5.13}$$

The modification to the s dependence comes from the derivatives acting on the α' in the exponent of s, so the first two terms in the expansion of f yield

$$\hat{A} \sim \left(1 + \frac{\alpha'(t_0 - t)}{c+1}\ln(\alpha's)\right)(\alpha's)^{\alpha_1(t)} \quad . \tag{5.14}$$

We thus see that the range of validity of α_1 as the effective Regge trajectory is extended from the region $t \geq t_0$ found in the previous subsection to the additional region (in $t < t_0$)

$$t_0 - t \ll \frac{c+1}{\alpha'\ln(\alpha's)} \quad . \tag{5.15}$$

6 Hard scattering limit

We begin by writing a tree Neveu-Schwarz amplitude for massless vectors

$$A_4^{(0)}(\alpha') = \left(\alpha'\right)^2 K \frac{\Gamma(-\alpha's)\,\Gamma(-\alpha't)}{\Gamma(1 - \alpha's - \alpha't)} \quad , \tag{6.1}$$

with the usual kinematical factor K (see, e.g., [24]). In general, a modified amplitude will have a subset of poles of the primary amplitude (6.1), if α' is replaced with α'/a_n such that the function a_n takes only positive integer values.[12] Take, for example, a polynomial of degree k with positive integer coefficients $P_k(n)$. According to our ansatz (3.4), the modified amplitude is then

$$\hat{A}_4^{(0)} = \sum_{n=1}^{\infty} w_n\, A_4^{(0)}(\alpha'/P_k) \quad . \tag{6.2}$$

For what follows we assume that w_n is a product of power functions like $n^\delta P_k^\gamma$.

To evaluate the amplitude in the hard scattering limit, $s \to \infty$, s/t fixed, we first split the sum into two parts and then replace the second sum with an integral as

$$\hat{A}_4^{(0)} = \sum_{n=1}^{[n_c/N]} w_n A_4^{(0)}(\alpha'/P_k) + \int_{[n_c/N]}^{\infty} dn\, w_n A_4^{(0)}(\alpha'/P_k) \quad , \tag{6.3}$$

where n_c is a solution of equation $P_k(n) = \alpha's$. For this value of n the arguments of gamma functions are of order 1, so Stirling formula is not applicable. Note that $n_c \sim \sqrt[k]{\alpha's}$ for $\alpha's \to \infty$. $[x]$ means the integer part of x. N is a free parameter such that Stirling formula is applicable for all the terms of the sum. If so, then the sum provides exponential falloff in the hard scattering limit. To see that the integral provides the desired power law, it is enough to rescale n as

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[12]This is not quite the same as Eq.(5.1): $t_0 = 0$ and $\alpha_0 = 1$. However, in the hard scattering limit it doesn't matter. We will have more to say on this subject below.

$n \to \sqrt[k]{\alpha's}\, n$. Indeed, in the lower integration limit a factor $\sqrt[-k]{\alpha's}$ cancels out the leading one from n_c. So, it behaves as $const + O(1/\sqrt[k]{\alpha's})$. As to the integrand, we have

$$\alpha_n(s) = \frac{1}{c_k n^k}\Big(1 - \frac{c_{k-1}}{c_k n \sqrt[k]{\alpha's}} + \dots\Big) \quad . \tag{6.4}$$

Finally, the amplitude behaves as

$$\hat{A}_4^{(0)} \sim (\alpha's)^{2+\gamma+(\delta+1)/k}\Big(1 + O\big(1/\sqrt[k]{\alpha's}\big)\Big) \quad . \tag{6.5}$$

We have used that $K \sim s^2$ in the hard scattering limit.[13]

   To compare with hard processes in QCD, we note that the corrections to the scaling behavior correspond to sea quarks and go as $1/s$. Thus, it seems to be reasonable taking the polynomial in the following form $P_k(n) = c_k n^k + c_0$. The other parameters can be fixed by noting that QCD amplitudes scale as $s^{2-n/2}$, where n is a total number of valence quarks. We take the option $\delta = -1$ and $\gamma = -n/2$ (see (3.5)). A significant difference from others is that ours is universal for all values of k.

   It is also worth looking at a pole structure of the amplitude (6.2). Using $\delta = -1$ and $\gamma = -n/2$, we find

$$\hat{A}_4^{(0)} = (\alpha')^2 K \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \frac{1}{n} P_k^{-1-n/2} \frac{1}{\alpha's - mP_k} \frac{\big(1+\alpha't/P_k\big)_{m-1}}{m!} \quad , \tag{6.6}$$

where $(x)_n$ stands for a Pochhammer polynomial. This equation shows that the poles are indeed a subset of those of the primary amplitude (6.1) and their distribution is a function of two integers (n, m). The residue of $\hat{A}_4^{(0)}$ at $\alpha's = l$ is given by

$$\gamma(l) = \frac{\alpha'}{t} K \sum_{\{n\}} \frac{1}{nl} P_k^{1-n/2} B^{-1}(\alpha't/P_k, l/P_k) \quad , \tag{6.7}$$

where $\{n\}$ is a set of integer solutions of the equation $l = m(c_k n^k + c_0)$. If the solutions don't exist, then $\gamma \equiv 0$.

   A pole at $l = 0$ is special because all primary amplitudes contribute. From this point of view it can be called infinitely degenerate, while all others as finitely degenerate. The residue is

$$\gamma(0) = \frac{\alpha'}{t} K \sum_{n=1}^{\infty} \frac{1}{n} P_k^{-n/2} \tag{6.8}$$

which is finite for positive n as it should be.[14] This pole corresponds to a massless ground state similar to that of the primary amplitude. The first massive state is due to a pole at $l = c_k + c_0$. Note that one can change its mass by varying the parameters c_k, c_0 but keeping α' close to the Planck length. The effect is similar to that of [11]. This gives a hint that spacetime geometry of our models might be warped.

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[13]As in QCD [25], this is due to scattering of vector particles.

[14]Note that in order that the sum be convergent a_n must increase for large n (see Eq.(3.2)).

So far we have made the simplest modification $\alpha' \to \alpha'/a_n$ of the first amplitude. The reason for doing so is that the slope is a dimensionful parameter which is easy to trace. On the other hand, the intercept is dimensionless, which makes it impossible to trace in kinematical factors K.[15] To bypass the K's, without losing generality consider the bosonic Lovelace-Shapiro amplitude [5, 26]

$$A_4^{(0)}(\alpha(s), \alpha(t)) = \frac{\Gamma\big(1 - \alpha(s)\big)\Gamma\big(1 - \alpha(t)\big)}{\Gamma\big(1 - \alpha(s) - \alpha(t)\big)} \quad , \tag{6.9}$$

where $\alpha(x) = \alpha_0 + \alpha' x$. Formula (3.4) then requires

$$\hat{A}_4^{(0)} = \sum_{n=1}^{\infty} w_n \, A_4^{(0)}(\alpha_n(s), \alpha_n(t)) \quad , \tag{6.10}$$

where $\alpha_n(x)$ is given by Eq.(3.2). This amplitude has a subset of poles of the primary amplitudes if and only if a_n and b_n take positive integer values. It seems natural to specialize to polynomials with integer coefficients, say, $a_n = P_k(n)$ and $b_n = P_{k'}(n)$ whose degrees are k and k', respectively. As to w_n, we take it as a product of power functions $w_n = n^\delta P_k^\gamma P_{k'}^\lambda$.

To evaluate the amplitude in the hard scattering limit we proceed as before. So, we first split the sum into two parts and then trade a second sum for an integral [16]

$$\hat{A}_4^{(0)} = \sum_{n=1}^{[n_c/N]} w_n A_4^{(0)}\big(\alpha_n(s), \alpha_n(-s\cos^2\phi/2)\big) + \int_{[n_c/N]}^{\infty} dn \, w_n A_4^{(0)}\big(\alpha_n(s), \alpha_n(-s\cos^2\phi/2))\big) \quad , \tag{6.11}$$

where n_c is a solution of equation $P_k(n) = \alpha' s$. N is a free parameter such that Stirling formula is applicable for all the terms of the sum. Thus the sum provides exponential falloff. To evaluate the integral we rescale n as $n \to \sqrt[k]{\alpha' s}\, n$. In the lower integration limit a factor $\sqrt[-k]{\alpha' s}$ cancels out the leading one of n_c. For the integrand, we obtain

$$\alpha_n(s) = \frac{1}{c_k n^k}\left(1 - \frac{c_{k-1}}{c_k n \sqrt[k]{\alpha' s}} + \dots\right)\left(1 + \frac{\alpha_0}{\alpha' s} - c_{k'} n^{k'} (\alpha' s)^{\frac{k'-k}{k}}\left(1 + \frac{c_{k'-1}}{c_{k'} n \sqrt[k']{\alpha' s}} \dots\right)\right) \quad . \tag{6.12}$$

Since n is bounded from below and $\alpha' s$ is large, we may treat subleading terms as corrections. As noted above, the corrections to the scaling behavior in QCD go as $1/s$. It follows that rational powers are not allowed. If so, then $c_{k-1} = \dots = c_1 = 0$, $c_{k'-1} = \dots = c_{1'} = 0$, and $k = k'$. In other words, both polynomials look very similar: they contain only the leading and constant terms and have the same degree. As a consequence, we recover Eq.(3.3) as expected.

Finally, we have

$$\hat{A}_4^{(0)} \sim \big(\alpha' s\big)^{\gamma + \lambda + (\delta+1)/k}\left(1 + O\big(1/\alpha' s\big)\right) \quad . \tag{6.13}$$

By comparison with the known results of [6] we fix $\delta = -1$ and $\gamma + \lambda = 2 - n/2$. Since the form of the polynomials is very restricted it makes no difference if we take w_n in the form $n^{-1} P_k^{2-n/2}$ (see (3.5)).

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[15] A related reason is that α' may be associated with a background metric, while it is unclear with which backgrounds may be associated α_0. One could think it of as a modulus corresponding to a ground state mass. In subcritical strings, the intercept may be related to other factors, such as the spacetime dimension, or coefficients of Liouville terms, which may in turn be related to a background "tachyon".

[16] Note that in the center of mass frame, $t \approx -s\cos^2\frac{\phi}{2}$ and $u \approx -s\sin^2\frac{\phi}{2}$.

7 Hadronic mass relations

A concrete, spectacular success of the early days of dual resonance models is that of [26, 27]. Combining the Veneziano type formulae for scattering amplitudes with the Adler condition, they found many mass relations that agree well with experiment. It seems natural to check whether the models of interest allow those relations too.

We begin by discussing $\pi\pi$ scattering along the lines of [26]. Consider the amplitude (6.10). The Adler condition requires the amplitude to vanish when $s = t = u = m_\pi^2 \approx 0$. Assuming that there is no cancellation between different terms, we get from the denominators

$$\alpha_n(0) = \frac{1}{2} \quad . \tag{7.1}$$

The novelty is the n-dependence. For the trajectory with n-independent intercept like (5.1) with $t_0 = 0$, the n-dependence is in fact missing. As a result, the trajectory obeys this requirement as in the usual case, i.e., if $\alpha_0 = 1/2$. It gives the intercept of the ρ trajectory. For the trajectory like (3.2), we conclude that

$$\alpha_0 = \frac{1}{2} a_n + b_n \quad . \tag{7.2}$$

Since α_0 does not depend on n, Eq.(7.2) shows that a_n and b_n must be integers of opposite signs. If so, the Adler condition provides the constraint on b_n. Inserting it back into Eq.(3.2) we get the trajectory discussed before.

We should caution the reader that in principle the amplitude can take the form $\hat{A}_4^{(0)} = (s + ct)f(s,t)$ with $f(0,0) \neq 0$. In this case the above derivation of the intercept fails.

It is straightforward to extend the above analysis to the case when all particles but one to be arbitrary hadrons $\pi + A \to B + C$ [27]. Assuming that amplitudes receive contributions from only one family of trajectories in each channel, the amplitude to be considered is given by Eq.(6.10) with $\alpha_n(s)$ and $\alpha_n(t)$ replaced by $\alpha_n^X(s)$ and $\alpha_n^Y(s)$. Here X and Y mean the corresponding families. The rest of the analysis goes along the lines of [27]. Thus, we get

$$a_n^A = a_n^X \tag{7.3}$$

and

$$\alpha_n^X(0) - \alpha_n^A(0) = \frac{1}{2} N_{AA} \quad , \tag{7.4}$$

with some integer N_{AA}. For the trajectories (5.1) with $t_0 = 0$, Eqs.(7.3)-(7.4) show that the two trajectories must have the same slopes, and intercepts which differ by a half-odd integer.[17] As a consequence, all the mass relations of [27] hold. For the trajectories (3.2), Eq.(7.4) provides the constraint on the b_n^I's. One possibility to resolve it is to take b_n^I in the form $b_n^I = \alpha_0 - \tilde{\alpha}_0 a_n^I$ that immediately leads to the trajectories (5.1) with $\alpha_0 \to \tilde{\alpha}_0$. Unfortunately, we do not know all the solutions of the constraint, so we can not answer whether all the trajectories reduce to those of (5.1).

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[17] Note that in this case N_{AA} is always an odd integer as in [27].

8   Further Issues

We begin with a special class of the trajectories (3.2). It is given by

$$a_n = n \quad , \quad b_n = B(n-1) \quad , \quad \alpha_0 = B \quad , \tag{8.1}$$

where B is an integer. To make one of the possible physical interpretations of this class somewhat clear, let us note that the effective tension of the nth term in the series (3.4)

$$T_n = Tn \quad , \quad T = 1/2\pi\alpha'$$

is nothing else but the tension of n fundamental strings. If so, one can think of the series as an expansion in fundamental strings. After this is understood, it immediately comes to mind to consider more complicated bound states. As is usual [28], this can be done by introducing D-strings.

In the presence of bound states (n, m) (n F-strings and m D-strings) it seems natural to modify the expression (3.4) as [18]

$$\hat{A} = \sum_{n=0, \, m=0}^{\infty}{}' \; w_{nm} \, A(n,m) \quad , \tag{8.2}$$

where the effective tension of the $(n, m)^{th}$ term is now

$$T_{nm} = T\sqrt{n^2 + \frac{m^2}{g^2}} \quad .$$

g stands for the string coupling. There is, however, a subtle point here: according to section 3 a_{nm} and b_{nm} must be integers. A possible way to avoid this difficulty is to take the original a_n and b_n as even-degree polynomials and restrict g to rational values. Since m^2/g^2 must be integer, it will restrict possible values of m in the sum (8.2). We will not drill deeper into details leaving them for future study.

A final remark: one surprise of $SL(2, Z)$-covariant superstrings is that the theory is in fact 12 dimensional [30]. It lives in a flat space with a diagonal metric taking values ± 1. As known, one may think of the model (8.1) as a zero-mode approximation to string theory whose spacetime metric is warped. For example, it is given by

$$ds^2 = f(r)dx^2 + dr^2 + ds_X^2 \quad , \tag{8.3}$$

where X is a five dimensional compact space. It seems natural to suggest that for the models (8.2) the corresponding metric is given by

$$ds^2 = f(r, \bar{r})dx^2 + dr^2 + d\bar{r}^2 + ds_{X'}^2 \quad , \tag{8.4}$$

where X' is now a six dimensional compact space. Note that the novelty is warping.

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[18]Interestingly enough, scattering amplitudes of $SL(2, Z)$-covariant superstrings as suggested in [29] are given by (8.2) with $w_{nm} = const.$

9 Conclusions

One question we have not addressed is the usual constraints at the string-loop level on the (critical) spacetime dimension and form of the trajectories (e.g., intercept). There might also be constraints already at the tree level, as we have not yet examined the higher-point amplitudes.

The asymptotic flatness of the top trajectories for large negative argument suggests a possible physical interpretation of the intercept: If these trajectories turn flat at $t = 0$ (as in AdS/CFT inspired models for the pomeron), then the intercept is related to the effective spin at $t = -\infty$. If the "state" corresponding to $t = -\infty$ is identified with a jet, and this effective spin with the parton carrying almost all the energy, then we expect intercept $1/2$ for the reggeon (spin of that quark) and intercept 1 for the pomeron (spin of that gluon), in qualitative agreement with experiment. (For the reggeon case corrections can be attributed to quark masses; for the pomeron case there can be significant corrections due to cuts.) In this picture it is a jet, the experimental signature of the parton, that is treated as "fundamental" rather than the corresponding parton itself: The jet is just a string in a certain off-shell kinematic limit.

These models might also be used for fundamental strings, including gravity. The existence of parton behavior at high energies indicates the graviton would be a bound state in a way similar to hadrons in QCD, so that gravity would disappear at short distances once the plasma phase is reached.

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